Direct Fuzzy Model-Reference Adaptive Control

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Intelligent systems may be viewed as a framework for solving the problems of nonlinear system control. The intelligence of the system in the nonlinear or changing environment is used to recognize in which environment the system currently resides and to service it appropriately. This paper presents a general methodology of adaptive control based on multiple models in fuzzy form to deal with plants with unknown parameters which depend on known plant variables. We introduce a novel model-reference fuzzy adaptive control system which is based on the fuzzy basis function expansion. The generality of the proposed algorithm is substantiated by the Stone-Weierstrass theorem which indicates that any continuous function can be approximated by fuzzy basis function expansion. In the sense of adaptive control this implies the adaptive law with fuzzified adaptive parameters which are obtained using Lyapunov stability criterion. The combination of adaptive control theory based on models obtained by fuzzy basis function expansion results in fuzzy direct model-reference adaptive control which provides higher adaptation ability than basic adaptive-control systems. The proposed control algorithm is the extension of direct model-reference fuzzy adaptive-control to nonlinear plants. The direct fuzzy adaptive controller directly adjusts the parameter of the fuzzy controller to achieve approximate asymptotic tracking of the model-reference input. The main advantage of the proposed approach is simplicity together with high performance, and it has been shown that the closed-loop system using the direct fuzzy adaptive controller is globally stable and the tracking error converges to the residual set which depends on fuzzification properties. The proposed approach can be implemented on a wide range of industrial processes. In the paper the foundation of the proposed algorithm are given and some simulation examples are shown and discussed. © 2002 Wiley Periodicals, Inc.

1. INTRODUCTION

The nature of dynamical systems usually implies slow changes of systems parameters and changes of the parameters due to the different operating conditions or operating points. In that case an adaptive controller should be designed to follow the changes of operating conditions and adapt in a certain prescribed way.

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INTERNATIONAL JOURNAL OF INTELLIGENT SYSTEMS, VOL. 17, 943–963 (2002)

Published online in Wiley InterScience (www.interscience.wiley.com). • DOI 10.1002/int.10054

The model-reference adaptive control systems are proven to be globally stable under certain assumptions on the unknown process: phase minimality; no disturbance or unmodeled dynamics; linearity; time invariance; and knowledge of the process relative ignore degree and the sign of so-called high-frequency gain. Unfortunately, the assumptions given above are often violated in practice and "adaptive algorithms as published in the literature are likely to produce unstable control systems if they are implemented on physical systems directly as they appear in the literature."[1] Many of the above assumptions can be circumvented on the cost of more complex adaptive and control laws. The robustness of the adaptive systems to unmodeled dynamics and bounded disturbance is treated in Refs. 2 and 3. Nonlinear adaptive control has been widely studied in the last decade,[4] but the results obtained seem not to be easily transferred to the engineering society since they require a fairly good knowledge of mathematics. Thus these approaches are avoided by practicing engineers.

In recent years, a lot of effort has been applied to neuro-fuzzy identification of complex plants, which cannot be easily modeled theoretically. The neuro-fuzzy adaptive control approaches based on neuro-fuzzy presentation of plant dynamics appeared in the literature in Ref. 5, where detailed discussion on identification and control of dynamical systems based on neural networks is given. This was also discussed in Ref. 6, where the tracking performance of model reference adaptive control using multilayer neural networks based on Lyapunov stability approach is studied, and in Refs. 7 and 8, where stable adaptive fuzzy controller for nonlinear systems is designed and explained. In Ref. 9 an adaptive control using multiple models is developed and investigated.

Our approach is qualitatively different. The model of the plant is given by the simple fuzzy Takagi-Sugeno model, which is very well suited for use in practice since it only requires basic knowledge of algebra. All equations are shown in simple form as in the description of the adaptive control of the linear plant. The basic idea of model-reference adaptive control is to introduce a global stability criterion into the design procedure and to choose the adaptive control law in such a way that the requirements of the stability criterion are fulfilled.[10] In our paper, an extension of model reference adaptive control to nonlinear systems is made using fuzzy sets theory. We are introducing a novel globally stable model-reference fuzzy adaptive control. The algorithm is based on direct model-reference adaptive control obtained by Lyapunov criterion. The main idea of our approach is fuzzification of adaptive parameters. The parameters are fuzzified corresponding to the process input, output, or state variables of the process. The paper is focused only on the problem of nonlinearity. All other typical problems which are common to adaptive systems in general can be treated and solved as proposed in the literature.[3,11,12]

The direct fuzzy adaptive controller directly adjusts the parameter of the fuzzy controller to achieve asymptotic tracking of the model-reference input.[13,14] It has been shown that asymptotic tracking convergence is possible if the approximation error is square integrable. This means that the fuzzy basis function expansion or fuzzy modeling of the plant dynamics should be designed in a way to achieve the modeling error (i.e., error between the real plant dynamics and the model) which is square integrable. Even in the case where the approximation (i.e., the modeling or approximation error) is not square integrable, we have shown that it is possible to achieve the asymptotic tracking of the model-reference signal. The main advantage of the proposed approach is simplicity together with high performance, and it has been proven that the closed-loop system using the direct fuzzy adaptive controller is globally stable and the tracking error converges to the residual set which depends on fuzzification properties. The proposed approach can be implemented on a wide range of industrial processes.

The paper is organized as follows: In Section 2 a description of the direct model-reference fuzzy adaptive system is given; in Section 3 two simulation examples are described and shown. In the conclusion some main observations are discussed.

2. DIRECT FUZZY MODEL-REFERENCE ADAPTIVE CONTROL

2.1. Model-Reference Adaptive Control of LTI Systems

The globally stable continuous model-reference adaptive control dynamics are given first. The goal of the model-reference adaptive system is to design a controller which forces the process to follow the model output, which is in the case of first-order plants, given by the following equation:

$$G_m(s) = \frac{y_m(s)}{w(s)} = \frac{b_m}{s + a_m} \tag{1}$$

where $w(t)$ stands for the reference signal and $y_m(t)$ for reference-model output. To obtain a perfect model, a prefilter with gain f and gain q in the feedback loop should be designed. Assuming the process transfer function

$$G_p(s) = \frac{y_p(s)}{u(s)} = \frac{b}{s + a} \tag{2}$$

and control law given in the following equation,

$$u = fw - qy_p \tag{3}$$

the closed-loop transfer function is given by

$$G_w(s) = \frac{fb}{s + a + bq} \tag{4}$$

The parameter errors between open-loop parameters and model-reference parameters are given next:

$$\begin{aligned} \tilde{b} &= fb - b_m \\ \tilde{a} &= a + bq - a_m \end{aligned} \tag{5}$$

Applying Equation 5 and subtracting the differential equations of the closed-loop system and reference model, the error equation is obtained:

$$\dot{e} + a_m e = \tilde{b}w - \tilde{a}y_p \tag{6}$$

where e defines the error between the plant output and model-reference response

$$e = y_p - y_m \tag{7}$$

Introducing a Lyapunov function

$$V(e, \tilde{a}, \tilde{b}) = e^2 + \frac{1}{\gamma_f}\tilde{b}^2 + \frac{1}{\gamma_q}\tilde{a}^2 \tag{8}$$

which is definite in $\mathcal{R}^3$, the space $(e, \tilde{a}, \tilde{b})$. The time derivative of $V(e, \tilde{a}, \tilde{b})$ along Equation 6 is given by:

$$\dot{V}(e, \tilde{a}, \tilde{b}) = -2a_m e^2 + \frac{2}{\gamma_f}\tilde{b}\dot{\tilde{b}} + 2\tilde{b}we + \frac{2}{\gamma_q}\tilde{a}\dot{\tilde{a}} - 2\tilde{a}y_p e \tag{9}$$

The condition $\dot{V}(e, \tilde{a}, \tilde{b}) \leqslant 0$ leads to the adaptive-control laws in Equation 10:

$$\dot{f} = -\frac{\gamma_f}{b}ew$$

$$\dot{q} = \frac{\gamma_q}{b}ey_p \tag{10}$$

where time-invariance of the plant is assumed. If the sign of the so-called high-frequency gain b is known in advance, Equation 10 can be rewritten as:

$$\dot{f} = -\gamma_f^* sign(b)ew$$
$$\dot{q} = \gamma_q^* sign(b)ey_p \tag{11}$$

where γ_f^* and γ_q^* are arbitrary positive constants.

Global stability is obtained in a small operation region where the process can be sufficiently described by a linear model. Problems arise in the case of unmodeled dynamics and nonlinear process plants. Our main motivation was to find a simple solution for adaptive control of nonlinear processes.

2.2. DFMRAC of Nonlinear Systems

The algorithm of Direct Fuzzy Model-Reference Adaptive Control (DFMRAC) will be presented next. Because of the simplicity, we are assuming the first-order model plant, but the approach can easily extend to a higher order.

The proposed fuzzy adaptive-control system assumes the fuzzification of forward gain $\boldsymbol{f}$ and feedback gain $\boldsymbol{q}$. The choice of fuzzification variables depends on the process behavior and is a similar problem to that of structural identification in the case of Takagi-Sugeno (TS) model.[15] The fuzzified gains are described by means of fuzzy numbers $\boldsymbol{f}$ and $\boldsymbol{q}$:

$$\boldsymbol{f}^T = [f_1, f_2, \ldots, f_k]$$

$$\boldsymbol{q}^T = [q_1, q_2, \ldots, q_k] \tag{12}$$

where k stands for number of fuzzy rules.

We assume that the process under investigation can be modeled by the TS fuzzy model of the form where two variables are given in the premise:

$$\mathbf{R}^i : \textbf{if } z_1 \textbf{ is } \mathbf{A}_i \textbf{ and } z_2 \textbf{ is } \mathbf{B}_i \textbf{ then } \dot{y}_p = -a_i y_p + b_i u, \qquad i = 1, \ldots, k \tag{13}$$

where z_1 and z_2 are the variables which influence mostly the nonlinear behavior of the process, y_p is the process output, $\dot{y}_p$ is the derivative of the process output and is at the same time the fuzzy model output, and A_i and B_i are fuzzy membership functions where $i_a = 1, \ldots, n_a$ and $i_b = 1, \ldots, n_b$. The number of membership functions for the first and the second input variables defines the number of rules $k = n_a \times n_b$. The membership functions have to cover the whole operating area of the closed-loop system. Using this type of TS fuzzy model and the models of higher order in consequent parts of the rules, a huge number of industrial processes can be modeled. The output of the TS model is then given by the following equation:

$$\dot{y}_p = \frac{\sum_{i=1}^{k} (\beta_i^*(\varphi(k))(-a_i y_p + b_i u))}{\sum_{i=1}^{k} \beta_i^*(\varphi(k))} \tag{14}$$

where $\varphi(k)$ represents the regressor which consists of input and output signals. The degree of fulfillment $\beta_i^*(\varphi(k))$ is obtained using T-norm which is in this case a simple algebraic product:

$$\beta_i^*(\varphi(k)) = T(\mu_{A_i}(z_1), \mu_{B_i}(z_2)) = \mu_{A_i}(z_1) \cdot \mu_{B_i}(z_2) \tag{15}$$

where $\mu_{A_i}(z_1)$ and $\mu_{B_i}(z_2)$ stand for degrees of fulfillment of the corresponding membership functions. The degrees of fulfillment for the whole set of rules can be written as:

$$\boldsymbol{\beta}^* = [\beta_1^*, \beta_2^*, \ldots, \beta_k^*]^T \tag{16}$$

and given in normalized form as

$$\boldsymbol{\beta} = \frac{\boldsymbol{\beta}^*}{\sum_{i=1}^{k} \beta_i^*} = [\beta_1, \beta_2, \ldots, \beta_k]^T \tag{17}$$

resulting in equality:

$$\sum_{i=1}^{k} \beta_i = 1 \tag{18}$$

Due to Equations 14 and 17, the process can be modeled in fuzzy form as:

$$\dot{y}_p = -\boldsymbol{\beta}^T \boldsymbol{a} y_p + \boldsymbol{\beta}^T \boldsymbol{b} u \tag{19}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ stand for fuzzified parameters of the process which have constant elements:

$$\boldsymbol{a}^T = [a_1, a_2, \ldots, a_k]$$
$$\boldsymbol{b}^T = [b_1, b_2, \ldots, b_k] \tag{20}$$

The model in Equation 19 can be viewed as a linear model with variable parameters which are called the global linear parameters and are given in the following equations:

$$\hat{a}(\varphi(k)) = \boldsymbol{\beta}^T\boldsymbol{a}$$
$$\hat{b}(\varphi(k)) = \boldsymbol{\beta}^T\boldsymbol{b} \tag{21}$$

In the Stone-Weierstrass theorem[16] it is shown that the general form of TS fuzzy model represents a universal approximator of any nonlinear dynamic system by fuzzy basis function expansion with arbitrary precision. The proposed structure of the TS fuzzy model is simplified, but it can be used for a wide range of industrial processes.

To develop the DFMRAC algorithm the control law given in Equation 22 is assumed first:

$$u = \frac{1}{\boldsymbol{\beta}^T\boldsymbol{b}}(\boldsymbol{\beta}^T\boldsymbol{f}^*w - \boldsymbol{\beta}^T\boldsymbol{q}^*y_p) \tag{22}$$

where $\boldsymbol{f}^*$ and $\boldsymbol{q}^*$ are control parameters to be obtained by adaptive law. The control law given in Equation 22 is in general not applicable because the unknown high-frequency gain of vector $\boldsymbol{b}$. Further development will result in an applicable control law. Implementing the control law in the basic closed loop of the control system, the following differential equation is obtained:

$$\dot{y}_p = -\boldsymbol{\beta}^T\boldsymbol{a}y_p + \boldsymbol{\beta}^T\boldsymbol{f}^*w - \boldsymbol{\beta}^T\boldsymbol{q}^*y_p \tag{23}$$

The fuzzy reference-model parameters $\boldsymbol{a}_m$ and $\boldsymbol{b}_m$ can be defined as:

$$\boldsymbol{a}_m = [a_m, a_m, \ldots, a_m]^T$$
$$\boldsymbol{b}_m = [b_m, b_m, \ldots, b_m]^T \tag{24}$$

and the reference model is written in the form of a fuzzy model by the following equation:

$$\dot{y}_m = -\boldsymbol{\beta}^T\boldsymbol{a}_m y_m + \boldsymbol{\beta}^T\boldsymbol{b}_m w \tag{25}$$

Subtracting the differential equation with fuzzy parameters in Equation 25 from Equation 23 the following error model is obtained:

$$\dot{e} + \boldsymbol{\beta}^T\boldsymbol{a}_m e = \boldsymbol{\beta}^T(\boldsymbol{f}^* - \boldsymbol{b}_m)w - \boldsymbol{\beta}^T(\boldsymbol{q}^* + \boldsymbol{a} - \boldsymbol{a}_m)y_p \tag{26}$$

The following equations are written to simplify further derivation:

$$\hat{\boldsymbol{b}} = \boldsymbol{f}^* - \boldsymbol{b}_m$$
$$\hat{\boldsymbol{a}} = \boldsymbol{q}^* + \boldsymbol{a} - \boldsymbol{a}_m \tag{27}$$

Implementing expressions from Equation 27 to Equation 26, the simplified equation is given by:

$$\dot{e} + \boldsymbol{\beta}^T \boldsymbol{a}_m e = \boldsymbol{\beta}^T \tilde{\boldsymbol{b}} w - \boldsymbol{\beta}^T \tilde{\boldsymbol{a}} y_p \tag{28}$$

Introducing the Lyapunov function:

$$V(e, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}) = e^2 + \sum_{i=1}^{k} \frac{1}{\gamma_{bi}} \tilde{b}_i^2 + \sum_{i=1}^{k} \frac{1}{\gamma_{ai}} \tilde{a}_i^2 \tag{29}$$

the time derivative of the function is given by:

$$\dot{V}(e, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}) = 2e\dot{e} + 2 \sum_{i=1}^{k} \frac{1}{\gamma_{bi}} \dot{\tilde{b}}_i \tilde{b}_i + 2 \sum_{i=1}^{k} \frac{1}{\gamma_{ai}} \dot{\tilde{a}}_i \tilde{a}_i \tag{30}$$

From $V(e, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}) > 0$ and $\dot{V}(e, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}) \leqslant 0$ the following equation should be fulfilled:

$$ew \sum_{i=1}^{k} \beta_i \tilde{b}_i + \sum_{i=1}^{k} \frac{1}{\gamma_{bi}} \dot{\tilde{b}}_i \tilde{b}_i = 0$$

$$ey_p \sum_{i=1}^{k} \beta_i \tilde{a}_i + \sum_{i=1}^{k} \frac{1}{\gamma_{bi}} \dot{\tilde{a}}_i \tilde{a}_i = 0, \qquad i = 1, \ldots, k \tag{31}$$

Equation 31 leads to the adaptive laws which are obtained in the following equation:

$$\dot{\tilde{b}}_i = -\gamma_{bi} e w \beta_i$$

$$\dot{\tilde{a}}_i = \gamma_{ai} e y_p \beta_i, \qquad i = 1, \ldots, k \tag{32}$$

Derivation of Equation 27 results in the following equations:

$$\dot{\tilde{\boldsymbol{b}}} = \dot{\boldsymbol{f}}^*$$

$$\dot{\tilde{\boldsymbol{a}}} = \dot{\boldsymbol{q}}^* \tag{33}$$

where time invariance of the controlled system is again assumed.

Assuming Equations 32 and 33, the following equations are obtained:

$$\dot{f}_i^* = -\gamma_{bi} e w \beta_i$$

$$\dot{q}_i^* = \gamma_{ai} e y_p \beta_i, \qquad i = 1, \ldots, k \tag{34}$$

The fuzzified vectors $\boldsymbol{f}^*$ and $\boldsymbol{q}^*$ are defined as:

$$\boldsymbol{f}^* = [f_1^*, \ldots, f_k^*]^T$$

$$\boldsymbol{q}^* = [q_1^*, \ldots, q_k^*]^T \tag{35}$$

The control law in Equation (22) is rewritten in the following form:

$$u = \boldsymbol{\beta}^T \boldsymbol{f} w - \boldsymbol{\beta}^T \boldsymbol{q} y_p \tag{36}$$

where $\boldsymbol{f}$ and $\boldsymbol{q}$ stand for:

$$\boldsymbol{f} = \frac{\boldsymbol{f}^*}{\boldsymbol{\beta}^T \boldsymbol{b}}$$

$$\boldsymbol{q} = \frac{\boldsymbol{q}^*}{\boldsymbol{\beta}^T \boldsymbol{b}} \tag{37}$$

According to Equations 34 and 37, the fuzzified adaptive parameters f_i and q_i, where $i = 1, \ldots, k$ are written as:

$$f_i = -\frac{\gamma_{bi}}{\boldsymbol{\beta}^T \boldsymbol{b}} \int_0^t e w \beta_i \, dt + f_i(0), \qquad \gamma_{bi} > 0$$

$$q_i = \frac{\gamma_{ai}}{\boldsymbol{\beta}^T \boldsymbol{b}} \int_0^t e y_p \beta_i \, dt + q_i(0), \qquad \gamma_{ai} > 0 \tag{38}$$

In the case of known high-frequency gain vector $\boldsymbol{b}$, which can be obtained by measuring the statical characteristics of the plant, the adaptive law in Equation 38 can be implemented directly. But, in general the high-frequency gain is not known in advance and the adaptive law in Equation 38 can not be implemented directly.

Next, we are going to investigate the adaptive law in Equation 38, and we will try to find the solution for the case of unknown high-frequency gain. According to the nature of the functions given in Equation 39, we can distinguish between two different examples of constant and nonconstant high-frequency gain vector $\boldsymbol{b}$:

$$\Gamma_{bi}(\boldsymbol{\beta}) = \frac{-\gamma_{bi}}{\boldsymbol{\beta}^T \boldsymbol{b}}$$

$$\Gamma_{ai}(\boldsymbol{\beta}) = \frac{\gamma_{ai}}{\boldsymbol{\beta}^T \boldsymbol{b}}. \tag{39}$$

Equation 39 is, in the case of constant high-frequency gain, simplified to:

$$\Gamma_{bi}(\boldsymbol{\beta}) = \gamma_f sign(b_i)$$

$$\Gamma_{ai}(\boldsymbol{\beta}) = \gamma_q sign(b_i), \qquad i = 1, \ldots, k \tag{40}$$

Assuming a high-frequency gain parameter of unknown constant value with a known sign of $sign(b_i) = sign(b_*)$, $i = 1, \ldots, k$. The fuzzy adaptive law of DFMRAC with constant high-frequency gain parameter is written in the integral form as follows:

$$\boldsymbol{f} = -\gamma_f sign(b_*) \int_0^t e w \boldsymbol{\beta} \, dt + \boldsymbol{f}(0), \qquad \gamma_f > 0$$

$$\boldsymbol{q} = \gamma_q sign(b_*) \int_0^t e y_p \boldsymbol{\beta}\, dt + \boldsymbol{q}(0), \qquad \gamma_q > 0 \tag{41}$$

and in differential form in Equation 42:

$$\dot{\boldsymbol{f}} = -\gamma_f sign(b_*) e w \boldsymbol{\beta}$$
$$\dot{\boldsymbol{q}} = \gamma_q sign(b_*) e y_p \boldsymbol{\beta} \tag{42}$$

In the case of nonconstant high-frequency gain the function in Equation 39 changes due to the change of $\boldsymbol{\beta}$. The parameters γ_{bi} and γ_{ai} are chosen arbitrarily. This means that those parameters can be chosen in a way so that they have a constant function in Equation 39 at the apexes of the membership functions. The difference between the value in apex and the value in the region between two apexes depends on fuzzification of the operating domain. When the fuzzification of the operating domain is sufficient, then the function in Equation 39 is approximately constant. In this case we assume a constant function in Equation 39 and introduce the leakage modification of fuzzy adaptive control laws as in the case of unmodelled dynamics. This modification is investigated next.

2.3. Stability Issues of DFMRAC

The fuzzy models are universal approximators of nonlinear processes.[17] The quality of approximation depends on fuzzification of the process state domain. The fuzzification means the number, type and distribution of membership functions.

It is not possible in general to find a vector $\boldsymbol{\Theta}^*$ that would permit zero tracking errors in each operating point since fuzzy modeling only guarantees arbitrary small tracking errors. This means that we should always take care with the unmodeled error. In the case of unmodeled dynamics the adaptive schemes may easily become unstable. The lack of robustness is primarily due to the adaptive law, which is nonlinear in general, and therefore more susceptible to modeling error effect.

The lack of robustness of adaptive law in Equation 42 in the presence of bounded disturbance can be solved by new approaches and adaptive laws which assure boundedness of all signals in the presence of plant uncertainties. This leads to a new body of work referred to as robust adaptive control.

To introduce the robustness into the adaptive control scheme, we convert the pure integral action of the adaptive law given by Equation 42 to a leaky integration. It is therefore referred to as the leakage modification.

In the case of unmodeled dynamics the dynamical normalization is used to obtain a robust solution. The design of the normalizing signal m will guarantee that $\frac{\eta_s}{m} \in \mathcal{L}_\infty$ and $\frac{\boldsymbol{\Psi}_f}{m} \in \mathcal{L}_\infty$, where η_s stands for modeling error term. The design of the dynamical normalization signal is the following:

$$m^2 = 1 + n_s^2$$
$$n_s^2 = m_s + \boldsymbol{\Psi}_f^T \boldsymbol{\Psi}_f + u^2 + y_p^2$$

$$\dot{m}_s = -\delta_0 m_s + u^2 + y_p^2$$
$$m_s(0) = 0 \tag{43}$$

where $\delta_0 > 0$ should be properly chosen.[18]

The idea of leakage modification is to modify the adaptive law so that the time derivative of the Lyapunov function used to analyze the adaptive scheme becomes negative in the space of the parameter when these parameters exceed certain bounds. This can be done by the modification of adaptive law presented in scalar form as:

$$\dot{f}_i = -\gamma_f sign(b_i)\epsilon w\beta_i - \gamma_f|\epsilon m|\nu_0 f_i\beta_i, \qquad i = 1, 2, \ldots, k$$
$$\dot{q}_i = -\gamma_q sign(b_i)\epsilon y_p\beta_i - \gamma_q|\epsilon m|\nu_0 q_i\beta_i, \qquad i = 1, 2, \ldots, k \tag{44}$$

or in vector form:

$$\dot{\boldsymbol{f}} = -\boldsymbol{\Gamma}_f\epsilon w\boldsymbol{\beta} - \boldsymbol{\Gamma}_f|\epsilon m|\nu_0\boldsymbol{F\beta}$$
$$\dot{\boldsymbol{q}} = \boldsymbol{\Gamma}_q\epsilon y_p\boldsymbol{\beta} - \boldsymbol{\Gamma}_q|\epsilon m|\nu_0\boldsymbol{Q\beta} \tag{45}$$

where we are assuming that $sign(b_i) = sign(b)$ for all $i = 1, \ldots, k$ and:

$$\boldsymbol{\Gamma}_f = \gamma_f sign(b)\boldsymbol{I}_{k\times k}$$
$$\boldsymbol{\Gamma}_q = \gamma_q sign(b)\boldsymbol{I}_{k\times k} \tag{46}$$

and $\boldsymbol{F}$ and $\boldsymbol{Q}$ stand for diagonal matrices where f_i, $i = 1, \ldots, k$ and q_i, $i = 1, \ldots, k$ are the diagonal elements.

The term $|\epsilon m|\nu_0$ in Equations 44 and 45 is called the leakage term. In the literature various choices[11,19] of the leakage term are known. The best results have been obtained by the upper choice which is called ϵ-modification introduced by Narendra and Annaswamy.[10] The constants in ϵ-modification are ν_0, which is a design constant, m is the normalizing signal, and ϵ is the normalized error between the reference-model output y_m and process output y_p.

In the compact matrix form the adaptive law is written as follows:

$$\dot{\boldsymbol{\Theta}} = \boldsymbol{\Gamma\Psi}_{f\epsilon} - \boldsymbol{\Gamma\Theta}_{diag}\boldsymbol{\beta}|\epsilon m|\nu_0 \tag{47}$$

where:

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_f & \boldsymbol{0}_{k\times k} \\ \boldsymbol{0}_{k\times k} & \boldsymbol{\Gamma}_q \end{bmatrix} \tag{48}$$

and:

$$\boldsymbol{\Theta}_{diag} = \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{Q} \end{bmatrix} \tag{49}$$

The stability properties of adaptive laws with the ϵ-modification and dynamic normalization guarantee that:

$$\epsilon,\ \epsilon n_s,\ \boldsymbol{\Theta},\ \dot{\boldsymbol{\Theta}} \in \mathscr{L}_\infty$$

$$\epsilon,\ \epsilon n_s,\ \dot{\boldsymbol{\Theta}} \in \mathcal{S}(\nu_0 + \eta_s^2/m^2) \tag{50}$$

and if n_s, $\boldsymbol{\Psi}_f \in \mathcal{L}_\infty$ and $\boldsymbol{\Psi}_f$ is persistently exciting and is independent of η_s, then $\boldsymbol{\Theta}$ converges exponentially to the residual set[18]

$$\mathcal{D}_\epsilon = \{\boldsymbol{\Theta} \in \mathcal{R}^n,\ |\boldsymbol{\Theta}| \leqslant c(\nu_0 + \bar{\eta})\} \tag{51}$$

where $c \leqslant 0$ and $\bar{n} = \sup_t \frac{\eta_s}{m}$. From Equation 50 it is also clear that ϵ, ϵn_s, and $\dot{\boldsymbol{\Theta}}$ are bounded in the $\mathcal{L}_{2e}$ sense and belong to the μ-small in the mean square sense.

3. SIMULATION EXAMPLES OF MODEL-REFERENCE FUZZY ADAPTIVE CONTROL FOR NONLINEAR PLANTS

Novel fuzzy adaptive control has been tested by means of simulation. Two different nonlinear process models were used, namely, the surge tank process model and the pH process.

3.1. Example: Level Control in a Surge Tank

The first simulation experiment was made on a surge tank,[20] which exhibits a strong nonlinear behaviour. A surge tank is a tank with parabolic profile. The goal is to control the liquid level, $h(t)$, in the tank by input flow $u(t)$. The differential equation describing this system is:

$$\dot{h} = \frac{-c\sqrt{2gh}}{S(h)} + \frac{1}{S(h)}u \tag{52}$$

$$S(h) = ah^2 + b \tag{53}$$

where $u(t)$ is the input flow or control input, which can be positive or negative (i.e., the system can both pull liquid out of the tank and put it in) $h(t)$ is liquid level (i.e., the output of the plant) and $S(h(t))$ is the cross-section area of the tank, $g = 9.8$ m s^{-2} is gravity, and $c = 1$ m^2 is the known cross-section area of the output pipe. The nominal plant parameters $a = 1$ and $b = 2$ m^2 are unknown. We assume that the control input is saturated at ± 20 m^3 s^{-1}. The reference-model transfer function is given as:

$$G_m(s) = \frac{1}{10s + 1} \tag{54}$$

The simulated fuzzy adaptive control system in the case of the level control of the surge tank is based on the adaptive law in Equations 45 and 47, where $\nu_0 = 0.000001$ and the adaptive gains are $\gamma_f = 2$, $\gamma_q = 2$. The signals are normalized instantaneously with normalizing signal m, defined in Equation 43 where normalizing parameter $\alpha = 5$ was used. Due to the nature of the nonlinearity, the operating domain of the process is divided into 27 subspaces, the variable $h(t)$ being divided into nine membership functions and the derivative $\dot{h}(t)$ into three.

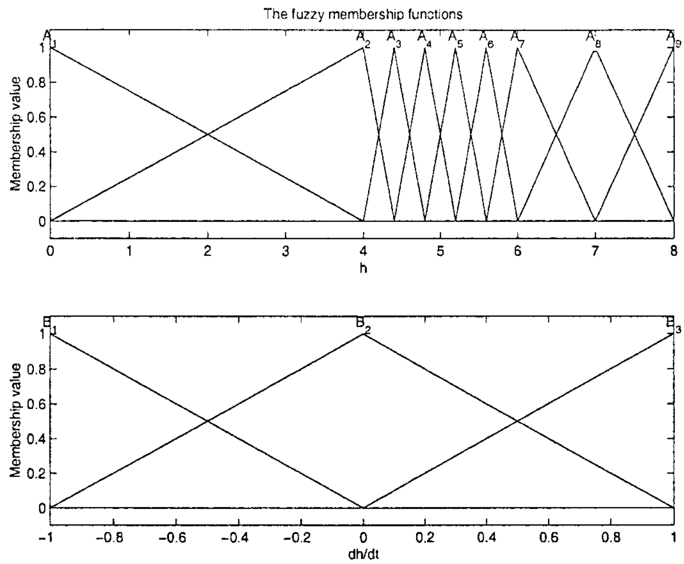

Figure 1. The fuzzy membership functions of (a) the process output signal and (b) the derivative of the process output signal.

Both linguistic variables (i.e., their membership functions) are shown in Figure 1. This means that the vector of degrees of fulfillment $\boldsymbol{\beta}$ depends on variables $h(t)$ and $\dot{h}(t)$ and is written as $\boldsymbol{\beta}(h, \dot{h})$. The derivative of the output signal is obtained using state variable filter $G_f(s) = s/s+1$. The results of the proposed fuzzy adaptive control system are shown in Figure 2. In Figure 3 the fuzzified feedforward $\boldsymbol{f}$ and feedback gain $\boldsymbol{q}$ are shown. The convergence of fuzzy adaptive parameters compared to those of the basic adaptive control system, which are presented in Figure 5, is slower due to the higher number of adapted parameters. But the convergence region of the tracking error is much smaller as it is shown in Figure 2 for fuzzy and in Figure 4 for the basic adaptive system. The comparison between output signals of fuzzy and the basic adaptive system and the model-reference signal in the case of surge tank level control is shown in Figure 6 and the control signals are depicted in Figure 7. The tracking errors are shown in Figure 8. It can be seen that the classical approach never converges because of the nonlinear nature of the controlled process. In the case of the fuzzy adaptive control scheme the tracking error depends on fuzzification of the operating domain and can be arbitrarily small.

3.2. Example: Adaptive Control of pH-Process

The problem of pH-process control is a common problem in many practical areas, including waste water treatment, biotechnology processing, and chemical processing.[21] There are two common characteristics of pH-control: (i) difficulties in pH-process control arising mainly from its heavy nonlinearity and uncertainty;

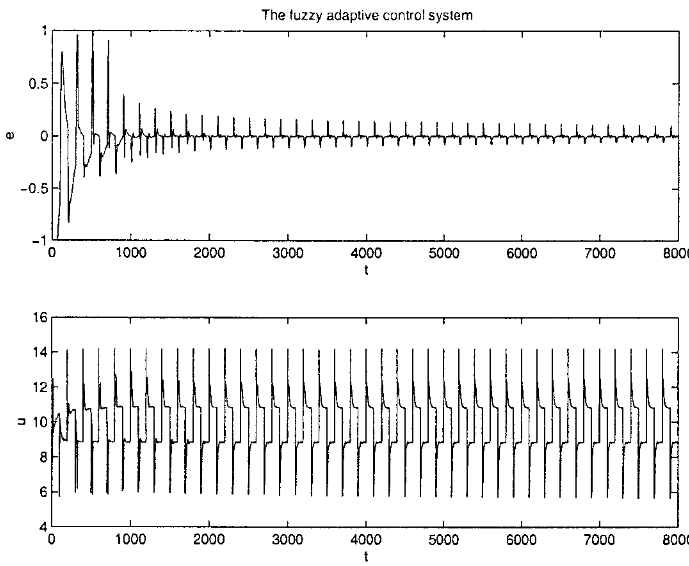

Figure 2. (a) The tracking error of the fuzzy adaptive system; (b) the control signal.

(ii) diversity in control approaches applied, ranging from simple PID control, adaptive control, nonlinear linearization control, gain-scheduling control and various types of model-based control to modern control systems, based on fuzzy systems and neural networks. One reason for the increasing number of papers in

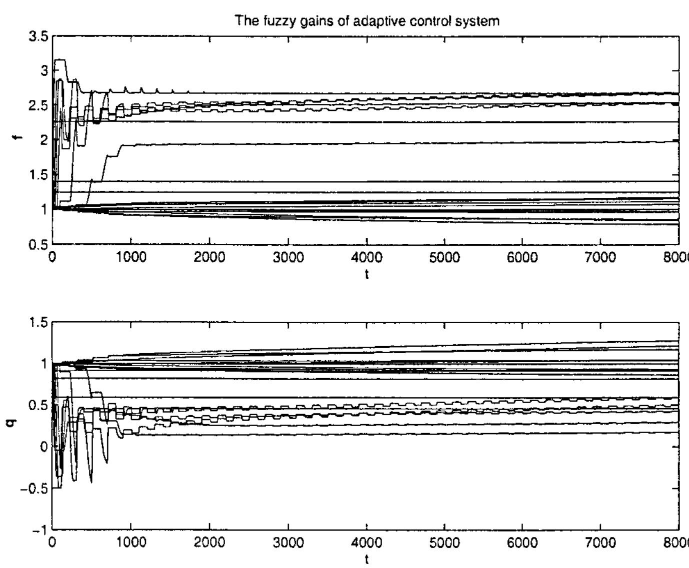

Figure 3. (a) The fuzzified feedforward gain $\boldsymbol{f}$; (b) the fuzzified feedback gain $\boldsymbol{q}$.

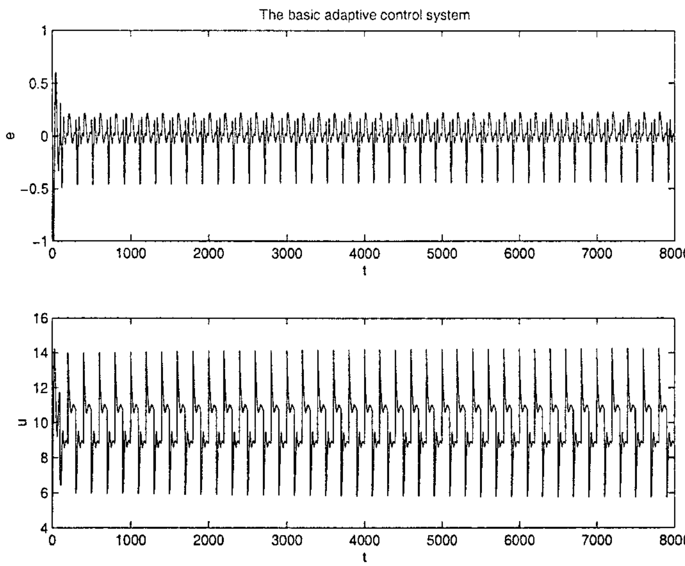

Figure 4. (a) The tracking error of the basic adaptive system; (b) the control signal.

recent years is the highly nonlinear character that makes pH control suitable for illustrating new nonlinear approaches. Another reason for such attention in the literature is the fact that practical pH-control has not yet been finally solved.

A simplified schematic diagram of the simulation test bench scale pH neu-

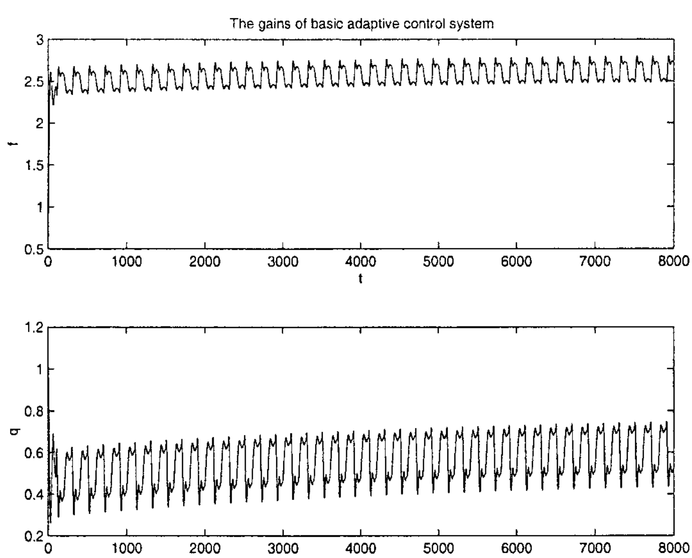

Figure 5. (a) The feedforward gain f; (b) the feedback gain q.

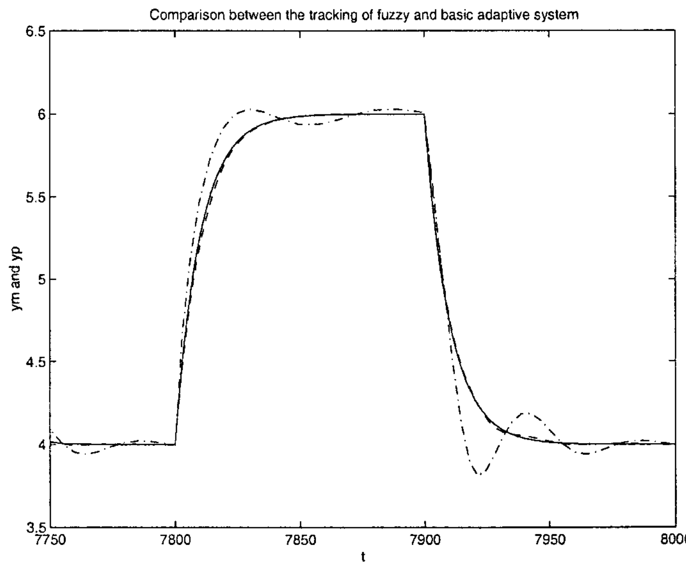

Figure 6. The comparison between output signals of fuzzy (——) and the basic adaptive system (—.) and the model-reference signal.

tralization tank is shown in Figure 9. Here, we are looking at a case where a strong acid is neutralized by a strong base in water in the presence of carbonate (buffer), and the output of the process is the effluent pH value. A dynamical model of

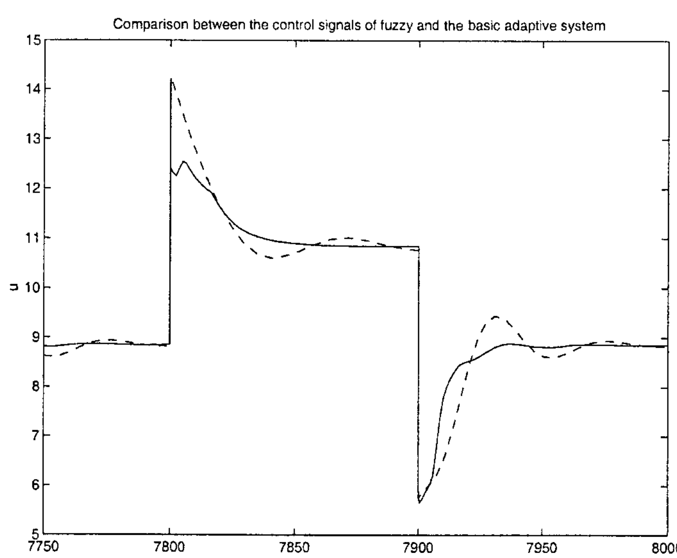

Figure 7. The comparison between control signals of fuzzy and the basic adaptive system (——).

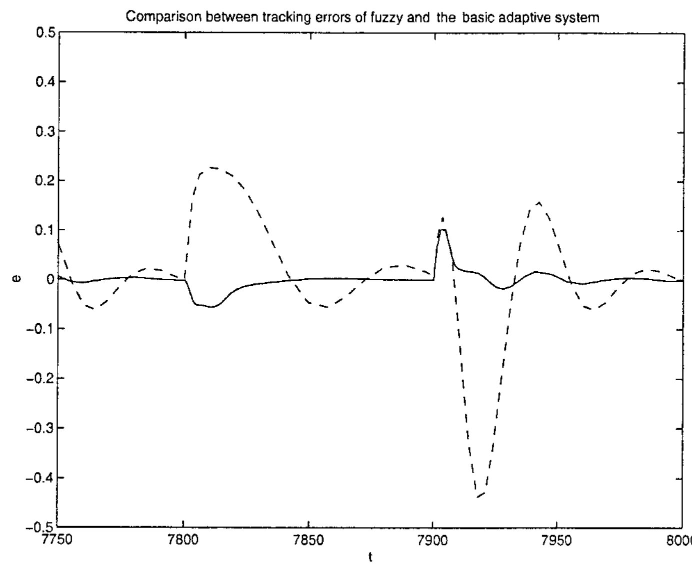

Figure 8. The comparison between fuzzy and basic adaptive system tracking errors (——).

pH-process has been derived using conservation equations and equilibrium relations employing a concept known as reaction invariant. Modeling assumptions include perfect mixing, constant density, fast reactions, and completely soluble ions. While keeping acid stream F_a (HNO_3) and buffer stream F_{bf} ($NaHCO_3$) at constant rate, the control objective is to follow some desired pH trajectory by manipulating the base stream F_b (NaOH and traces of $NaHCO_3$).

The chemical equilibrium is obtained by defining two reaction invariants for each inlet and outlet stream ($i \in [1, 4]$):

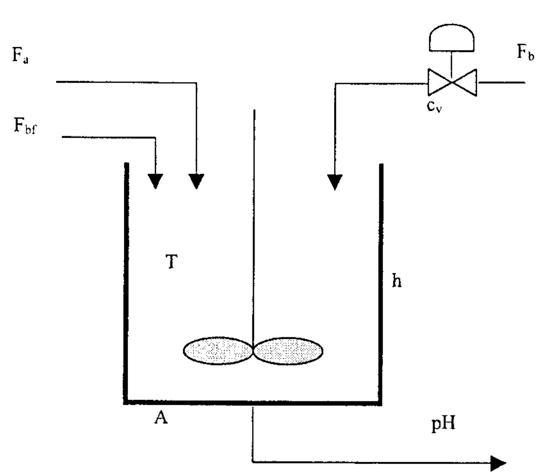

Figure 9. The pH-process.

$$W_{ai} = [H^+]_i - [OH^-]_i - [HCO_3^-]_i - 2[CO_3^{2-}]_i$$
$$W_{bi} = [H_2CO_3]_i + [HCO_3^-]_i + [CO_3^{2-}]_i \tag{55}$$

where the invariant W_{ai} is a charge-related quantity, while W_{bi} equals the concentration of the $[CO_3^{2-}]$ ions. Unlike pH, these invariants are conserved quantities. Using the equilibrium constants:

$$K_{a1} = [HCO_3^-][H^+][H_2CO_3]^{-1}$$
$$K_{a2} = [CO_3^{2-}][H^+][H_2CO_3^-]^{-1}$$
$$K_w = [H^+][OH^-] \tag{56}$$

an implicit relation for $[H^+]$ can be derived:

$$W_a + \frac{K_w}{[H^+]} + W_b \frac{\dfrac{K_{a1}}{[H^+]} + \dfrac{2K_{a1}K_{a2}}{[H^+]^2}}{1 + \dfrac{K_{a1}}{[H^+]} + \dfrac{K_{a1}K_{a2}}{[H^+]^2}} - [H^+] = 0$$
$$\mathrm{pH} = -\log([H^+]) \tag{57}$$

Equation 57 defines the static titration curve function relation pH value to reaction variables.

The complete dynamical model is given by a mass balance equation and two differential equations for the effluent invariants:

$$A\dot{h} = F_a + F_{bf} + F_b - c_v\sqrt{h}$$
$$Ah\dot{W}_{a4} = F_a(W_{a1} - W_{a4}) + F_{bf}(W_{a2} - W_{a4}) + F_b(W_{a3} - W_{a4})$$
$$Ah\dot{W}_{b4} = F_a(W_{b1} - W_{b4}) + F_{bf}(W_{b2} - W_{b4}) + F_b(W_{b3} - W_{b4}) \tag{58}$$

Nominal values of the parameters and operating condition are taken from Ref. 21. The acid and buffer flow-rates are supposed to be known and kept constant.

The simulated fuzzy adaptive control system in the case of pH process control is based on the adaptive law defined by Equations 45 and 47, where $\nu_0 = 0.0001$ and the adaptive gains are $\gamma_f = 2$, $\gamma_q = 2$. The signals are normalized dynamically with signal m defined by Equation 43, where parameter $\delta_0 = 0.05$ was used. The reason dynamical normalization is in unmodeled and unmeasured dynamics, is because the plant actually exhibits the third order dynamics which in our case is modeled with the first order fuzzy model. One way to assure a robust control is to apply a leakage integration and dynamic normalization of the signals.

The operating domain of the process is due to the nature of the nonlinearity divided according to the variable $y_p(t)$. But the fastest convergence was obtained when $y_m(t)$ was also included in the premise. Both linguistic variables, i.e. their membership functions, are shown in Figure 10. This means that the vector of degrees of fulfillment $\boldsymbol{\beta}$ depends on variable $y_p(t)$ and $y_m(t)$ and is written as $\boldsymbol{\beta}(y_p, y_m)$. This disagrees with the fuzzy model of the plant proposed, but gives better

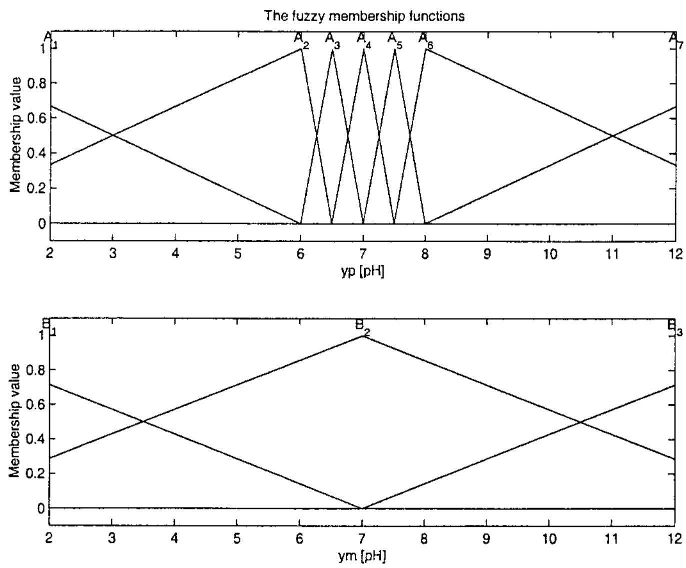

Figure 10. The fuzzy membership functions of (a) the process output signal and (b) the model-reference output.

results. The results of the proposed fuzzy adaptive control applied on pH process are presented in Figure 11, where the tracking error and control signal are shown. In Figure 12 the fuzzified feedforward ***f*** and feedback gain ***q*** are shown. In the end,

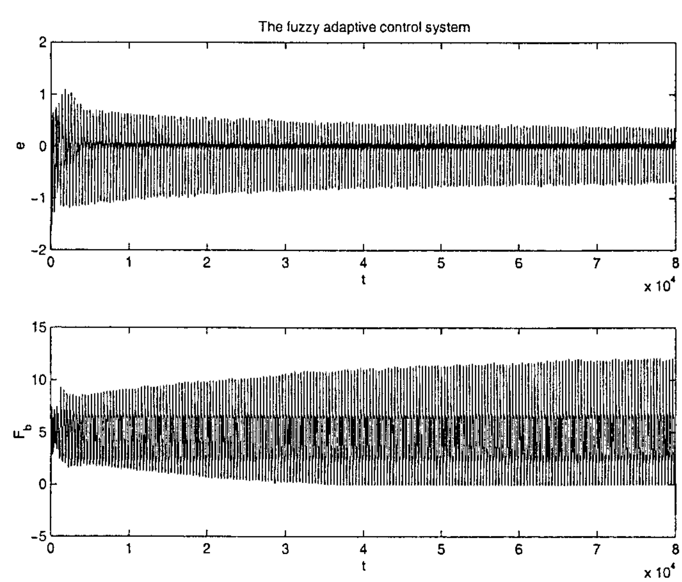

Figure 11. (a) The tracking error of the fuzzy adaptive system; (b) the control signal.

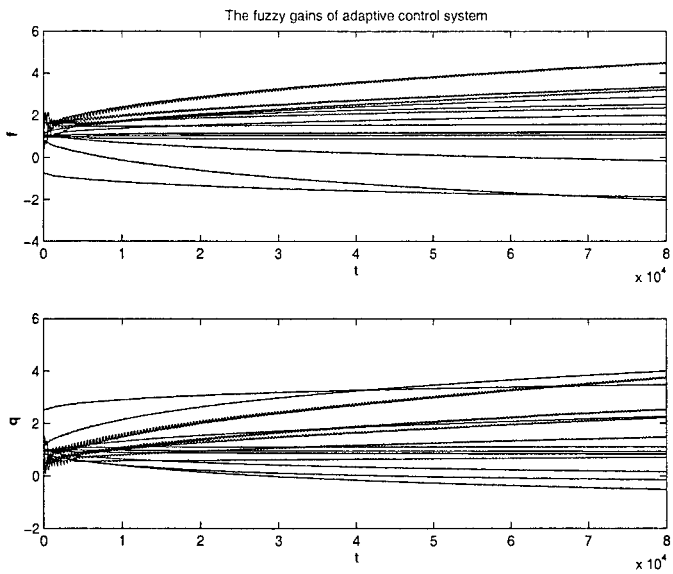

Figure 12. (a) The fuzzified feedforward gain ***f***; (b) the fuzzified feedback gain ***q***.

the tracking performance of the proposed fuzzy adaptive system in the case of pH control and the control signal, which is saturated between 0 and 20, are shown in Figure 13. In spite of highly nonlinear behavior and underestimated dynamics of

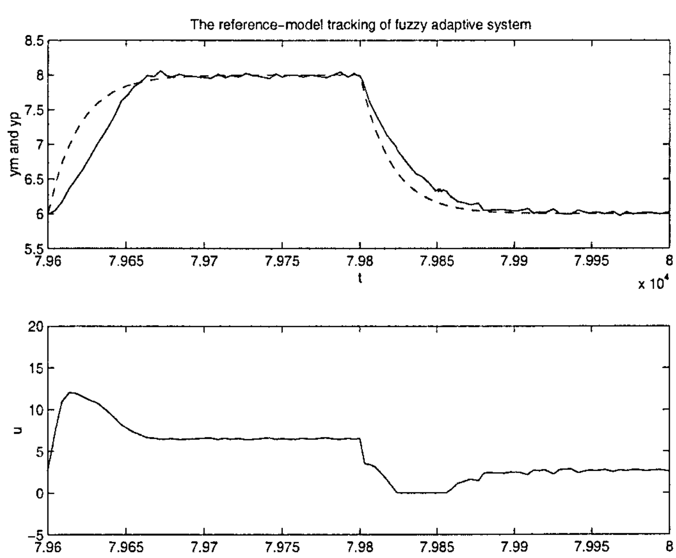

Figure 13. (a) The comparison between output signals of the fuzzy adaptive system and the model-reference signal (——); (b) the control signal.

the plant the obtained results are reasonably good. It should be pointed out that the system is robust and the parameters in Figure 12 converge to constant values. The envelope in Figure 11(a) also stays within an acceptable range of zero tracking error.

4. CONCLUSION

In this paper a novel model reference fuzzy adaptive control system is introduced. It is based on the Lyapunov stability criterion. The adaptive parameters of the system are fuzzified. The main goal of the proposed approach was the extension of globally stable adaptive control to nonlinear plants. The parameters are fuzzified corresponding to the process input, output, or state variables of the process. The development of the novel algorithm has been tested using simulation on different nonlinear systems, including unmodeled and unmeasured dynamics. The combination of adaptive-control theory based on models obtained by fuzzy basis function expansion results in fuzzy direct model-reference adaptive control which provides higher adaptation ability than basic adaptive control systems. The main advantage of the proposed approach is simplicity together with high performance. In this paper, the foundation for proposed algorithm is given and some simulation examples are shown and discussed.

Acknowledgement

The author would like to thank BMFT-KFA Jülich, Germany and the Ministry of Science, Slovenia which funded the author's stay at TU Darmstadt with Professor Isermann, where the main part of the research was done.

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